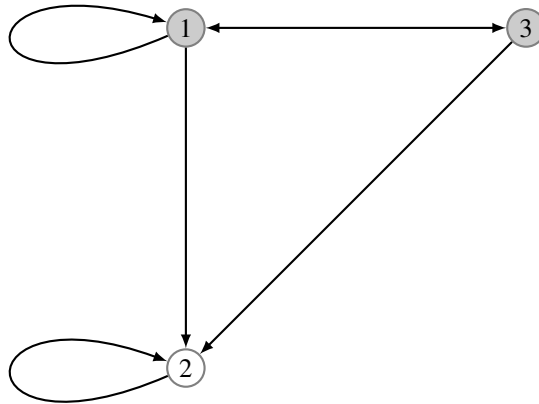


Homework exercises, Week 8, part b (due Tuesday 3 April).

1. Below, you are given a model with a unary predicate P and a binary predicate R . Shading marks objects with property P , \rightarrow indicates that two objects are R -related, a \leftrightarrow link indicates that the R -relation runs in both directions, and loops indicate that an object is R -related to itself.



Which of the following formulas are true in the model?

- (a) $\exists x(Px \wedge Rxx)$
- (b) $\forall x(Px \rightarrow Rxx)$
- (c) $\forall x(Px \rightarrow \exists yRxy)$
- (d) $\exists x(Px \wedge \neg Rxx)$
- (e) $\exists x(\exists yRyx \wedge Rxx)$

(f) $\forall x(\exists yRyx \rightarrow Rxx)$

(g) $\forall x(Rxx \rightarrow \exists yRxy)$

2. Use the tableau method to find out whether the arguments below are valid or invalid. If you find an argument to be invalid, try to use the information from one of the open branches to construct a counter-model.

(a) $\forall x\forall y(Rxy \rightarrow Ryx), Rab \models Rba$

(b) $\forall x\forall y(Rxy \rightarrow Ryx), Rab \models Raa$

3. Use the tableau method to find out whether the following formulas are satisfiable or not. If a formula is satisfiable, use the information from one of the open branches to construct a model that makes that makes the formula true.

(a) $\exists x\exists y\forall z((Rxz \wedge Ryz) \rightarrow Rzz)$

(b) $\exists x\exists y(\forall z(Rxz \wedge Ryz) \wedge \neg\exists u(Rxu \wedge Ryu))$

4. Use the tableau method to find out whether the following formulas are tautologies or not. If a formula is a not tautology, use the information from one of the open branches to construct a counter-model.

(a) $\forall x\exists y(Rxy \rightarrow Rxx)$

(b) $\exists xRxx \rightarrow \forall x\forall y\neg Rxy$

(c) $(\exists xRxa \wedge \exists xRxb) \rightarrow \forall x\forall yRxy$

(d) $\exists x\exists y(Rxy \wedge Ryx) \rightarrow \neg\forall x\neg\exists yRxy$