Amsterdam University College

Logic, Information flow and Argumentation

Homework exercises, Week 3, part a (due Friday 24 February).

1. Check whether the following formulas are logically equivalent. If not, specify a valuation V that satisfies one but not the other.
   (a) $p \vee q$ and $p \leftrightarrow q$,
   (b) $\neg(p \vee q)$ and $\neg p \wedge \neg q$,
   (c) $p \rightarrow q$ and $\neg p \vee q$,
   (d) $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$.
   (e) $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \vee r)$.
2. Suppose φ and ψ are logically equivalent sentences.
   (a) Show that φ is a tautology if and only if ψ is a tautology.
   (b) Show that φ is a contradiction if and only if ψ is a contradiction.
   (c) Show that φ is a tautology if and only if $\neg\psi$ is a contradiction.
   (d) Show that $\neg\varphi$ and $\neg\psi$ are logically equivalent.
   (e) Show that $(\varphi \leftrightarrow \psi)$ is a tautology.
3. (a) Suppose we add a new operator $\heartsuit$ to propositional logic, defined according to the following truth table:

φ	$\heartsuit$	ψ
1	1	1
0	1	1
1	0	0
0	1	0

   i. Define $\heartsuit$ in the standard language of propositional logic (i.e., write a sentence using the connectives $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ which is logically equivalent to "$\varphi \heartsuit \psi$".)

ii. Do the same for the fragment of standard propositional logic using only $\wedge$ and $\neg$.

iii. Do the same for the fragment of standard propositional logic using only $\vee$ and $\neg$.

(b) Same for the operator $\clubsuit$ defined by:

φ	$\clubsuit$	ψ
1	0	1
0	0	1
1	1	0
0	1	0

4. On slide 6 of week 3 you see all the possible logical expressions that can be defined in propositional logic. For each of them, what is the simplest formula you can find to express it? For example:
The meaning of φ_2 can be expressed by the formula $p \vee q$.
The meaning of φ_{12} can be expressed by the formula $p \wedge \neg q$.