



Logic, Information flow and Argumentation

Homework exercises, Week 2, part a (due Friday 17. February).

1. On Slides 4 and 5, you see how the *truth value* of each logical connective is defined (by the *truth tables*). To evaluate the truth of a complex formula containing more than one connective, you follow the construction tree of the formula to compute the truth values bottom up - you can see this on Slide 8. For example, suppose you want to evaluate the truth value of the expression “ $(p \vee q) \wedge (r \vee s)$ ”, assuming that p is true and q, r, s are false (i.e., the *valuation* V defined by $V(p) = 1, V(q) = 0, V(r) = 0, V(s) = 0$). Then proceed as follows:

	$(p$	\vee	$q)$	\wedge	$(r$	\vee	$s)$
	1		0		0		0
step 1:	1	1	0		0	0	0
step 2:	1	1	0	0	0	0	0

So the truth value of the entire expression is 0 under the given truth value of p, q, r, s (i.e., under the given valuation V).

Evaluate the truth of the following expressions in propositional logic, given the truth values of the atomic propositions:

- (a) $(\neg p \vee q)$, assuming that p and q are both true,
- (b) $p \vee (q \wedge (\neg p))$, assuming that p is false and q is true
- (c) $p \rightarrow (q \wedge (\neg q))$, assuming that p and q are both false
- (d) $(p \wedge q) \leftrightarrow (\neg r)$, assuming that p is true and q, r are false,
- (e) $(p \wedge q) \vee (r \wedge s)$, assuming that p, q are true and r, s are false,
- (f) $p \rightarrow (q \rightarrow (r \rightarrow s))$, assuming that p, q, r are true and s is false,
- (g) $\neg(p \rightarrow q) \vee (r \rightarrow s)$, assuming that p, q, r are true and s is false,

(h) $((p \rightarrow (q \vee r)) \rightarrow s) \wedge ((p \wedge q) \vee (r \rightarrow s))$, assuming that p, q, r, s are all false.

2. You can evaluate the truth of an expression in all possible situations (i.e., for all possible *valuations*, see Slide 9). For example, if you have two propositional variable p and q and want to evaluate the expression " $(p \vee q) \rightarrow p$ ", you do the same as in Exercise 1, for four different combinations of truth-value assignments to p and q (i.e., four different valuations):

$(p$	\vee	$q)$	\rightarrow	p
1	1	1	1	1
1	1	0	1	1
0	1	1	0	0
0	0	0	1	0

Evaluate the truth of the following expressions in all possible situations (for all possible valuations):

- (a) $p \vee (q \vee r)$,
- (b) $((p \rightarrow q) \rightarrow p) \rightarrow p$
- (c) $(p \wedge \neg p) \vee (p \rightarrow p)$,
- (d) $\neg(p \vee q) \leftrightarrow ((\neg p) \wedge (\neg q))$,